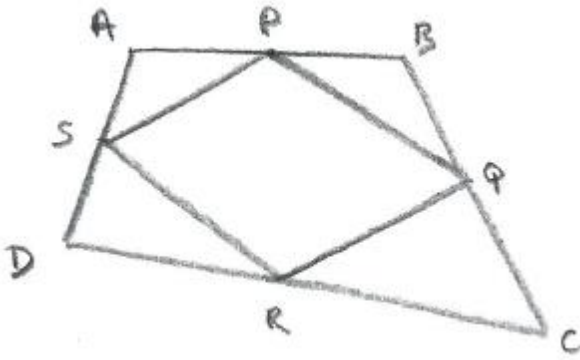


STEP/Vectors Q3 (30/6/23)

Use vectors to prove that the mid-points of the sides of any quadrilateral form the vertices of a parallelogram.

Solution



Referring to the diagram (where $\underline{a} = \overrightarrow{OA}$ etc),

$$\underline{q} - \underline{p} = \frac{1}{2}(\underline{b} + \underline{c}) - \frac{1}{2}(\underline{a} + \underline{b}) = \frac{1}{2}(\underline{c} - \underline{a})$$

$$\text{and } \underline{r} - \underline{s} = \frac{1}{2}(\underline{c} + \underline{d}) - \frac{1}{2}(\underline{a} + \underline{d}) = \frac{1}{2}(\underline{c} - \underline{a}) = \underline{q} - \underline{p}$$

So the sides PQ & SR are of equal length and parallel.

This means that $PQRS$ is a parallelogram.