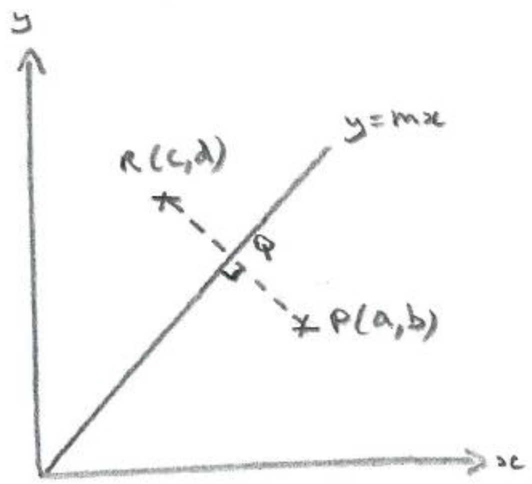
# STEP/Vectors Q2 (30/6/23)

Show that the coordinates of the reflection of the point $(a, b)$ in the line $y = mx$ are $\frac{1}{m^2+1}\begin{pmatrix} a(1-m^2)+2bm \\ 2am+b(m^2-1) \end{pmatrix}$

**Solution**




Referring to the diagram, let $\lambda\binom{1}{m}$ be the point Q.

Then, as $\overrightarrow{QP}$ is perpendicular to the line $y = mx$,

$\overrightarrow{QP}.\binom{1}{m} = 0$; ie $\binom{a-\lambda}{b-\lambda m}.\binom{1}{m} = 0$,

so that $a - \lambda + (b - \lambda m)m = 0$

$\Rightarrow \lambda(m^2 + 1) = a + bm$, and $\lambda = \frac{a+bm}{m^2+1}$

Then $\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR} = \lambda\binom{1}{m} + \overrightarrow{PQ}$

$= \lambda\binom{1}{m} + \binom{\lambda - a}{\lambda m - b}$

$= \binom{2\lambda - a}{2\lambda m - b}$

$= \frac{1}{m^2+1}\binom{2(a+bm) - a(m^2+1)}{2m(a+bm) - b(m^2+1)}$

$= \frac{1}{m^2+1}\binom{a(1-m^2) + 2bm}{2am + b(m^2-1)}$

[Note that, when $m = 1$, R is $(b, a)$.]