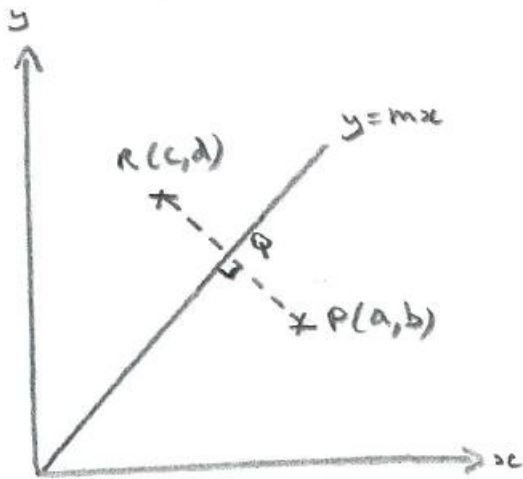


STEP/Vectors Q2 (30/6/23)

Show that the coordinates of the reflection of the point (a, b) in

the line $y = mx$ are $\frac{1}{m^2+1} \begin{pmatrix} a(1 - m^2) + 2bm \\ 2am + b(m^2 - 1) \end{pmatrix}$

Solution



Referring to the diagram, let $\lambda \begin{pmatrix} 1 \\ m \end{pmatrix}$ be the point Q.

Then, as \overrightarrow{QP} is perpendicular to the line $y = mx$,

$$\overrightarrow{QP} \cdot \begin{pmatrix} 1 \\ m \end{pmatrix} = 0; \text{ ie } \begin{pmatrix} a - \lambda \\ b - \lambda m \end{pmatrix} \cdot \begin{pmatrix} 1 \\ m \end{pmatrix} = 0,$$

$$\text{so that } a - \lambda + (b - \lambda m)m = 0$$

$$\Rightarrow \lambda(m^2 + 1) = a + bm, \text{ and } \lambda = \frac{a + bm}{m^2 + 1}$$

$$\text{Then } \overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR} = \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} + \overrightarrow{PQ}$$

$$= \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} + \begin{pmatrix} \lambda - a \\ \lambda m - b \end{pmatrix}$$

$$= \begin{pmatrix} 2\lambda - a \\ 2\lambda m - b \end{pmatrix}$$

$$= \frac{1}{m^2 + 1} \begin{pmatrix} 2(a + bm) - a(m^2 + 1) \\ 2m(a + bm) - b(m^2 + 1) \end{pmatrix}$$

$$= \frac{1}{m^2 + 1} \begin{pmatrix} a(1 - m^2) + 2bm \\ 2am + b(m^2 - 1) \end{pmatrix}$$

[Note that, when $m = 1$, R is (b, a) .]