STEP/Vectors Q1 (30/6/23)

Show that if $|\underline{a} - \underline{b}| = |\underline{a} + \underline{b}|$, then $\underline{a}$ & $\underline{b}$ are perpendicular.

Solution

$|\underline{a}-\underline{b}| = |\underline{a}+\underline{b}| \Rightarrow |\underline{a}-\underline{b}|^2 = |\underline{a}+\underline{b}|^2$

$\Rightarrow (\underline{a}-\underline{b}).(\underline{a}-\underline{b}) = (\underline{a}+\underline{b}).(\underline{a}+\underline{b})$

$[\ \underline{x}.\underline{x} = |\underline{x}|\ |\underline{x}| cos 0° = |\underline{x}|^2\ ]$

$\Rightarrow \underline{a}.\underline{a} - \underline{a}.\underline{b} - \underline{b}.\underline{a} + \underline{b}.\underline{b} = \underline{a}.\underline{a} + \underline{a}.\underline{b} + \underline{b}.\underline{a} + \underline{b}.\underline{b}$

$\Rightarrow -2\underline{a}.\underline{b} = 2\underline{a}.\underline{b}$ [since $\underline{a}.\underline{b} = \underline{b}.\underline{a}$]

$\Rightarrow \underline{a}.\underline{b} = 0$

and hence $\underline{a}$ & $\underline{b}$ are perpendicular

[Geometrically, $|\underline{a}-\underline{b}|$&$|\underline{a}+\underline{b}|$ are the 'short' and 'long' diagonals of the parallelogram formed from the adjacent sides $\underline{a}$ & $\underline{b}$. When these diagonals are equal, the parallelogram is a rectangle.]