STEP/Vectors: Exercises - Overview (30/6/23)

Q1

Show that if $|\underline{a} - \underline{b}| = |\underline{a} + \underline{b}|$, then $\underline{a}$ & $\underline{b}$ are perpendicular.

Q2

Show that the coordinates of the reflection of the point (a, b) in the line $y = mx$ are $\frac{1}{m^2+1}\begin{pmatrix} a(1 - m^2) + 2bm \\ 2am + b(m^2 - 1) \end{pmatrix}$

Q3

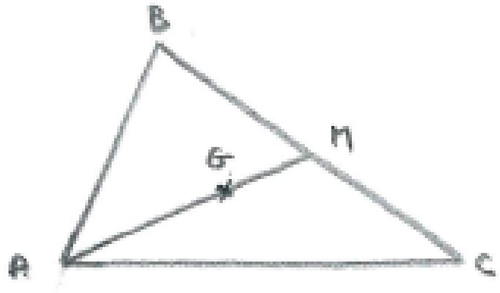
Use vectors to prove that the mid-points of the sides of any quadrilateral form the vertices of a parallelogram.

Q4

Prove that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians.

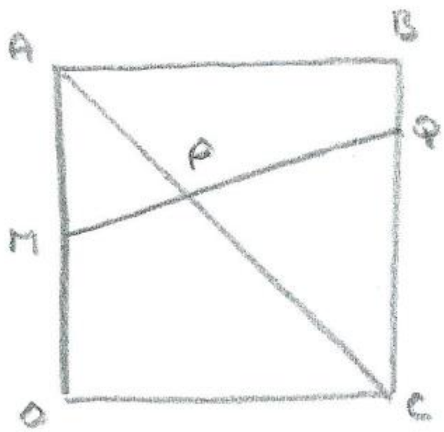
Q5

Given that the centre of mass of a triangular lamina lies 2/3 of the way along any of the medians, prove that it has position vector $\frac{1}{3}$ $(\underline{a} + \underline{b} + \underline{c})$.



Q6

In the diagram below, OABC is a square, M is the midpoint of OA, BQ is a quarter of BC, and P is the intersection of AC and MQ.



If $\underline{a} = \overrightarrow{OA}$ and $\underline{c} = \overrightarrow{OC}$, show that $\overrightarrow{OP} = \frac{3}{5}\underline{a} + \frac{2}{5}\underline{c}$

Q7

Find the angle between adjacent sloping faces of a right square-based pyramid, where the faces are equilateral triangles (as shown in Figure 1).

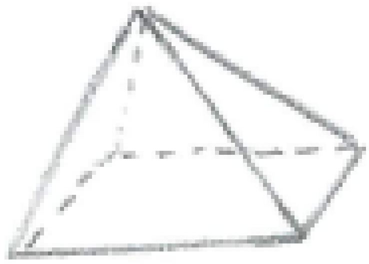


Figure 1

Q8

Given that $\boldsymbol{a}$, $\boldsymbol{b}$ & $\boldsymbol{c}$ are linearly independent vectors, establish whether the vectors $\boldsymbol{a}+\boldsymbol{b}$, $\boldsymbol{a}-\boldsymbol{c}$ & $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}$ are linearly independent.

Q9

Are the vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ & $\begin{pmatrix}0\\0\\0\end{pmatrix}$ linearly independent?