STEP/Trigonometry Q9 (30/6/23)

Show that $sec^2\theta(cosec\theta - sin\theta) \equiv cosec\theta$

Solution

Method 1

$LHS = cosec\theta sec^2\theta\left(1 - \frac{sin\theta}{cosec\theta}\right)$

[‘forcing’ the LHS into the required form; ie aiming to show that

$sec^2\theta\left(1 - \frac{sin\theta}{cosec\theta}\right) = 1$]

$= cosec\theta sec^2\theta(1 - sin^2\theta)$

$= cosec\theta sec^2\theta cos^2\theta = cosec\theta$, as required

Method 2

$sec^2\theta(cosec\theta - sin\theta) \equiv cosec\theta$ is equivalent to

$sec^2\theta(cosec\theta - sin\theta) - cosec\theta \equiv 0 \quad (1)$

And $(1) = cosec\theta(sec^2\theta - 1) - sec^2\theta sin\theta$

$= cosec\theta tan^2\theta - tan\theta sec\theta$

$= tan\theta(cosec\theta tan\theta - sec\theta)$

$= tan\theta sec\theta(cosec\theta sin\theta - 1)$

$= tan\theta sec\theta(1 - 1) = 0$, as required

Method 3

$sec^2\theta(cosec\theta - sin\theta) \equiv cosec\theta$ is equivalent to

$\frac{sec^2\theta(cosec\theta - sin\theta)}{cosec\theta} = 1 \quad (2)$

And $(2) = \frac{sec^2\theta(1 - sin^2\theta)}{1} = sec^2\theta cos^2\theta = 1$, as required