STEP/Trigonometry Q8 (30/6/23)

Show that $cos^4\theta + sin^4\theta = 1 - \frac{1}{2}sin^2(2\theta)$

Solution

Consider

$$1 = (cos^2\theta + sin^2\theta)^2 = cos^4\theta + sin^4\theta + 2cos^2\theta sin^2\theta$$

Then $cos^4\theta + sin^4\theta = 1 - 2cos^2\theta sin^2\theta = 1 - \frac{1}{2}(2cos\theta sin\theta)^2$

$= 1 - \frac{1}{2}sin^2(2\theta)$, as required.