STEP/Trigonometry Q7 (30/6/23)

Write $\sqrt{2(1-cos\theta)}$ and $\sqrt{2(1+cos\theta)}$ in the form $asin(b\theta)$ or $acos(b\theta)$

Solution

$cos(2\theta) = cos^2\theta - sin^2\theta = 1 - 2sin^2\theta,$

so that $1 - cos(2\theta) = 2sin^2\theta$

and hence $\sqrt{2(1 - cos\theta)} = 2\sin\left(\frac{\theta}{2}\right)$

Also, $cos(2\theta) = cos^2\theta - sin^2\theta = 2cos^2\theta - 1,$

so that $1 + cos(2\theta) = 2cos^2\theta$

and hence $\sqrt{2(1 + cos\theta)} = 2\cos\left(\frac{\theta}{2}\right)$