

STEP/Trigonometry Q6 (30/6/23)

Show that $\frac{\sec\theta+1-\tan\theta}{\sec\theta+1+\tan\theta} \equiv \sec\theta - \tan\theta$

Solution

To show that $\frac{\sec\theta+1-\tan\theta}{\sec\theta+1+\tan\theta} - (\sec\theta - \tan\theta) \equiv 0$:

$$\text{LHS} = \frac{(\sec\theta+1-\tan\theta) - (\sec\theta-\tan\theta)(\sec\theta+1+\tan\theta)}{\sec\theta+1+\tan\theta}$$

$$\text{Numerator} = (\sec\theta + 1 - \tan\theta)$$

$$- (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)$$

$$= (\sec\theta + 1 - \tan\theta) - (\sec^2\theta - \tan^2\theta) - (\sec\theta - \tan\theta)$$

$$= (\sec\theta + 1 - \tan\theta) - 1 - (\sec\theta - \tan\theta) = 0$$