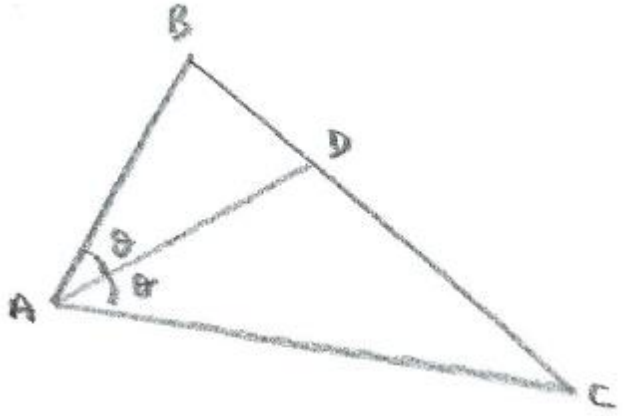
STEP/Trigonometry Q4 (30/6/23)

Angle Bisector Theorem

Referring to the diagram below, the Angle Bisector theorem says that $\frac{BD}{DC} = \frac{AB}{AC}$. Prove the Angle Bisector Theorem.



Solution

By the Sine rule for triangle ABD, $\frac{BD}{sin\theta} = \frac{AB}{sinADB}$ (1)

and, for triangle ADC, $\frac{DC}{sin\theta} = \frac{AC}{sinADC} = \frac{AC}{sinADB}$ (2)

Then (1) $\Rightarrow \frac{sin\theta}{sinADB} = \frac{BD}{AB}$ and (2) $\Rightarrow \frac{sin\theta}{sinADB} = \frac{DC}{AC}$

so that $\frac{BD}{AB} = \frac{DC}{AC}$

and hence $\frac{BD}{DC} = \frac{AB}{AC}$