

STEP/Trigonometry Q3 (30/6/23)

Solve $\sin\theta = \cos 4\theta$ for $0 < \theta < \pi$

Solution

$$\sin\theta = \sin\left(\frac{\pi}{2} - 4\theta\right)$$

$$\text{Hence } \theta = \frac{\pi}{2} - 4\theta + 2n\pi \quad (1) \quad \text{or } \theta = \left(\pi - \left[\frac{\pi}{2} - 4\theta\right]\right) + 2n\pi \quad (2)$$

$$\text{From (1), } 5\theta = \frac{\pi(1+4n)}{2}, \text{ so that } \theta = \frac{\pi(1+4n)}{10}$$

$$\text{giving } \theta = \frac{\pi}{10}, \frac{\pi}{2} \text{ or } \frac{9\pi}{10}$$

$$\text{From (2), } -3\theta = \frac{\pi(1+4n)}{2}, \text{ so that } \theta = \frac{-\pi(1+4n)}{6}$$

$$\text{giving } \theta = \frac{\pi}{2} \text{ again}$$

$$\text{Thus, the solutions are } \theta = \frac{\pi}{10}, \frac{\pi}{2} \text{ or } \frac{9\pi}{10}$$

A sketch confirms that these are plausible.

