STEP/Trigonometry Q1 (30/6/23)

How many solutions does the equation

$sin(2cos(2x) + 2) = 0$ have, for $0 \le x \le 2\pi$?

Solution

With $u = 2cos(2x) + 2, 0 \leq x \leq 2\pi \Rightarrow 2(-1) + 2 \leq u \leq 2(1) + 2$

ie $0 \leq u \leq 4$

Then $sin u = 0 \Rightarrow u = 0 \; or \; \pi$

$\Rightarrow cos(2x) = -1 \; or \; \dfrac{\pi-2}{2} = \dfrac{\pi}{2} - 1$

Now making the substitution $w = 2x, 0 \leq w \leq 4\pi$

Referring to the graph of $cos w$,

$cos w = -1$ has 2 solutions (for w), and $cos w = \dfrac{\pi}{2} - 1$ has 4 solutions; making 6 solutions in total.

As $x = \dfrac{w}{2}$, there are also 6 solutions for x.

[A variation on the above approach is to say that

$2cos(2x) + 2$ must equal $n\pi$, for suitable integer n

Then, either $n = 0$, with $cos(2x) = -1$,

or $n = 1$, with $cos(2x) = \dfrac{\pi}{2} - 1$

(no other values of n are consistent with $2cos(2x) + 2$),

as before.]