

STEP/Trigonometry Q13 (30/6/23)

Show that

(i) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

(ii) $\cos^4\theta + \sin^4\theta = 1 - \frac{1}{2}\sin^2(2\theta)$

Solution

$$(i) \cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) \\ = \cos 2\theta(1) = \cos 2\theta$$

(ii) Consider

$$1 = (\cos^2\theta + \sin^2\theta)^2 = \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta$$

$$\text{Then } \cos^4\theta + \sin^4\theta = 1 - 2\cos^2\theta\sin^2\theta = 1 - \frac{1}{2}(2\cos\theta\sin\theta)^2 \\ = 1 - \frac{1}{2}\sin^2(2\theta), \text{ as required.}$$