

STEP/Trigonometry Q12 (30/6/23)

Express $-\cos\theta$ in the form $\cos\alpha$ (where α is to be found in terms of θ), using an algebraic method.

Solution

$$\begin{aligned}-\cos\theta &= -\sin\left(\frac{\pi}{2} - \theta\right) = \sin\left(\theta - \frac{\pi}{2}\right) \\&= \cos\left(\frac{\pi}{2} - \left[\theta - \frac{\pi}{2}\right]\right) = \cos(\pi - \theta) \quad (\text{or } \cos(3\pi - \theta) \text{ etc})\end{aligned}$$

Alternatively, $-\cos\theta = -\cos(-\theta) = -\sin\left(\frac{\pi}{2} - [-\theta]\right)$

$$\begin{aligned}&= \sin\left(-\frac{\pi}{2} - \theta\right) = \cos\left(\frac{\pi}{2} - \left[-\frac{\pi}{2} - \theta\right]\right) = \cos(\pi + \theta) \\&\quad (\text{or } \cos(3\pi + \theta) \text{ etc})\end{aligned}$$