

STEP/Trigonometry Q10 (30/6/23)

Show that $\tan\theta + \cot\theta \equiv \sec\theta\operatorname{cosec}\theta$

Solution

Method 1

$\tan\theta + \cot\theta \equiv \sec\theta\operatorname{cosec}\theta$ is equivalent to

$$\tan\theta + \cot\theta - \sec\theta\operatorname{cosec}\theta \equiv 0 \quad (1)$$

$$\text{LHS of (1)} = \frac{\sin^2\theta + \cos^2\theta - 1}{\cos\theta\sin\theta} = \frac{0}{\cos\theta\sin\theta} = 0, \text{ as required}$$

Method 2

$\tan\theta + \cot\theta \equiv \sec\theta\operatorname{cosec}\theta$ is equivalent to

$$\frac{\tan\theta + \cot\theta}{\sec\theta\operatorname{cosec}\theta} = 1 \quad (2)$$

$$\text{LHS of (2)} = \frac{\sin^2\theta + \cos^2\theta}{1} = 1, \text{ as required}$$