# STEP/Sequences & Series Q4 (27/6/23)

Show that $\sum_{r=1}^{\infty} ra^r = \dfrac{a}{(1-a)^2}$

## Solution

$$\sum_{r=1}^{\infty} r a^r = a \frac{d}{da} \sum_{r=1}^{\infty} a^r = a \frac{d}{da}\left(\frac{a}{1-a}\right) \quad (\text{when } |a| < 1)$$

$$= a \cdot \frac{(1-a)-a(-1)}{(1-a)^2} = \frac{a}{(1-a)^2}$$