

**STEP/Sequences & Series Q1 (27/6/23)**

Triangular numbers are defined as follows:

$$T_r = \frac{1}{2}r(r + 1) \text{ for integer } r \geq 1$$

Prove that  $\sum_{r=1}^{\infty} \frac{1}{T_r} = 2$

**Solution**

$$\begin{aligned}\sum_{r=1}^{\infty} \frac{1}{T_r} &= 2 \sum_{r=1}^{\infty} \frac{1}{r(r+1)} = 2 \sum_{r=1}^{\infty} \left( \frac{1}{r} - \frac{1}{r+1} \right) \\ &= 2 \left\{ \left( \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots \right) - \left( \frac{1}{2} + \frac{1}{3} + \dots \right) \right\} = 2\end{aligned}$$