STEP/Sequences & Series: Exercises - Overview
(27/6/23)

Q1

Triangular numbers are defined as follows:

$T_r = \frac{1}{2}r(r+1)$ for integer $r \geq 1$

Prove that $\sum_{r=1}^{\infty} \frac{1}{T_r} = 2$

Q2

'Perfect powers' can be defined as follows:

m^k for integer $m \geq 2$ & integer $k \geq 2$

Prove that the sum of the reciprocals of all perfect powers is 1 (including duplicates; eg $4^2 = 2^4$).

Q3

Show that $\sum_{r=0}^{n} \binom{n}{r} = 2^n$

Q4

Show that $\sum_{r=1}^{\infty} ra^r = \frac{a}{(1-a)^2}$

Q5

(i) Consider the sequence defined by $u_n = au_{n-1} + b$,

where a & b are real constants, and u_0 is given.

What familiar sequences are special cases of this sequence?

(ii) Define a new sequence by $v_n = u_n + c$

For what value of c, in terms of a & b, will v_n be a geometric sequence? For what value of a does this not work?

(iii) If $u_n = 2u_{n-1} + 3$, and $u_0 = 4$, find a formula for u_n in terms of n

(iv) Find a similar formula for $u_n = au_{n-1} + b$, where u_0 is given.

(v) Under what conditions will u_n be constant? Give a non-trivial example.