Selections

(i) Number of ways of selecting r distinct items from n, if repetitions are allowed, and order is important

Solution

Number of ways of selecting r items from n, if repetitions are allowed, and order is important $= n^r$

(ii) Number of ways of selecting r items from n, if repetitions are not allowed, and order is important

Solution

Number of ways of selecting r items from n, if repetitions are not allowed, and order is important

$= n(n-1)\dots(n-[r-1]) = n(n-1)\dots(n-r+1)$

[known as a Permutation]

$P(n,r)$ or ${}^{n}P_{r} = \frac{n!}{(n-r)!} = n(n-1)\dots(n-r+1)$

(iii) Number of ways of selecting r items from n, if repetitions are not allowed, and order is not important

Solution

Number of ways of selecting r items from n, if repetitions are not allowed, and order is not important [known as a Combination]

$C(n, r)$ or ${}^nC_r$ or $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

[${}^nC_r$ can be obtained from ${}^nP_r = \frac{n!}{(n-r)!}$ by dividing by $r!$, to remove duplication (the ${}^nP_r$ ordered ways can be divided into groups of $r!$, containing the same items, but in a different order).]

(iv) Number of ways of selecting r items from n, if repetitions are allowed, and order is not important.

[Number of ways of selecting r items from n, if repetitions are allowed, and order is not important.]

Solution

eg $BBCE$ selected from $ABCDEF$ $(r = 4, n = 6)$

write as $|XX|X||X|$

(| indicates that we are moving on to the next letter, and XX indicates that we are selecting 2 items from the current letter: so $|XX|X||X|$ means: move on to B (without selecting any As); then select 2 Bs; then move on to the Cs; select 1 C; move on to D, and then on to E; select 1 E; then move on to F, but select no Fs)

$=$ Number of ways of choosing r positions for the Xs,

out of the $n-1$ |s and r Xs (giving a total of $n-1+r$)

$$= \binom{n-1+r}{r}$$

Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ [where $\binom{n}{r} \equiv {}^{n}C_{r}$]

Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

Solution

If r items are to be chosen from n items, then either the 1st item is included or it isn't.

If it is included, then there are $\binom{n-1}{r-1}$ ways of choosing the remaining $r-1$ items that are required.

If it isn't included, then there are $\binom{n-1}{r}$ ways of choosing the remaining r items that are required.

This gives a total of $\binom{n-1}{r-1} + \binom{n-1}{r}$ ways of choosing the r items.