STEP/Probability Q3 (12/6/23)

When choosing the venue for an international conference, 3 countries are shortlisted at random from a list of 9, of which 4 are European and 5 are from the rest of the world. What is the probability that at least 2 of the countries shortlisted are European?

Solution

Method 1a

Number of ways of selecting 3 countries out of 9

$$= \binom{9}{3} = \frac{9(8)(7)}{6} = 3(4)(7) = 84$$

Number of ways of selecting 2 European countries out of 4, and 1 non-European country out of 5 $= \binom{4}{2}\binom{5}{1} = 6(5) = 30$

Number of ways of selecting 3 European countries out of 4, and 0 non-European country out of 5 $= \binom{4}{3}\binom{5}{0} = 4(1) = 4$

Prob(at least 2 European countries) $= \frac{30+4}{84} = \frac{17}{42}$

Method 1b

Number of ways of selecting 1 European country out of 4, and 2 non-European countries out of 5 $= \binom{4}{1}\binom{5}{2} = 4(10) = 40$

Number of ways of selecting 0 European countries out of 4, and 3 non-European countries out of 5 $= \binom{4}{0}\binom{5}{3} = 1(10) = 10$

Prob(at least 2 European countries) $= 1 - \frac{40+10}{84} = 1 - \frac{25}{42} = \frac{17}{42}$

Method 2a

Prob(2 European countries)$= 3 \times$ Prob(EER) $= 3 \times \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7} = \frac{5}{14}$

(where EER means that the 1st & 2nd countries selected are European, and the 3rd is from the rest of the world)

Prob(3 European countries)$= \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21}$

Hence Prob(at least 2 European countries) $= \frac{5}{14} + \frac{1}{21} = \frac{15+2}{42} = \frac{17}{42}$

Method 2b

$$\text{Prob(0 European countries)} = \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{3 \times 2 \times 7} = \frac{5}{42}$$

Prob(1 European country)

$$= 3 \times \text{Prob(ERR)} \ = 3 \times \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{5 \times 2}{3 \times 7} = \frac{10}{21}$$

Hence Prob(at least 2 European countries) $= 1 - \frac{5}{42} - \frac{10}{21} = 1 - \frac{25}{42} = \frac{17}{42}$