# STEP/Probability Q1 (12/6/23)

Three numbers are chosen at random from the integers 1 to $n$ (without replacement). What is the probability that the highest number chosen is $k$? (where $3 \leq k \leq n$)

## Solution

## Method 1

Required prob. =

Prob(1st number chosen is $k$)

×Prob(2nd number chosen is smaller than $k$)

×Prob(3rd number chosen is smaller than $k$)

+Prob(1st number chosen is smaller than $k$)

×Prob(2nd number chosen is $k$)

×Prob(3rd number chosen is smaller than $k$)

+Prob(1st number chosen is smaller than $k$)

×Prob(2nd number chosen is smaller than $k$)

×Prob(3rd number chosen is $k$)

$$= \frac{1}{n} \times \frac{k-1}{n-1} \times \frac{k-2}{n-2} + \frac{k-1}{n} \times \frac{1}{n-1} \times \frac{k-2}{n-2} + \frac{k-1}{n} \times \frac{k-2}{n-1} \times \frac{1}{n-2}$$

$$= \frac{3(k-1)(k-2)}{n(n-1)(n-2)}$$

## Method 2

Required prob.

$$= \frac{no.\ of\ ways\ of\ choosing\ 3\ numbers, with\ k\ being\ the\ highest}{no.\ of\ ways\ of\ choosing\ 3\ numbers}$$

$$= \frac{\binom{k-1}{2}}{\binom{n}{3}} = \frac{\left(\frac{(k-1)(k-2)}{2!}\right)}{\left(\frac{n(n-1)(n-2)}{3!}\right)} = \frac{3(k-1)(k-2)}{n(n-1)(n-2)}$$