STEP/Probability: Exercises - Overview (12/6/23)

Q1

Three numbers are chosen at random from the integers 1 to n (without replacement). What is the probability that the highest number chosen is k? (where $3 \leq k \leq n$)

Q2

The probability that a (biased) coin shows Heads is p, and the probability that it shows Tails is q.

(i) Show that $pq \leq \frac{1}{4}$

(ii) Show that $p^3 + q^3 \geq \frac{1}{4}$

Q3

When choosing the venue for an international conference, 3 countries are shortlisted at random from a list of 9, of which 4 are European and 5 are from the rest of the world. What is the probability that at least 2 of the countries shortlisted are European?

Q4

An unbiased die has n sides, numbered 1 to n. If the die is thrown twice, find the probability that the score on the 2nd throw is greater than the score on the 1st throw.

Q5

In a simplified game of tennis, a player wins a game by being the first player to win 4 points (ie 15, 30, 40, Game). If the probability that player A wins each point is $\frac{2}{3}$, show that the probability that player A wins the game is $\frac{1808}{2187}$