

STEP/Polynomials Q4 (26/6/23)

What is the minimum value of $(x^2 - 4x + 3)(x^2 + 4x + 3)$, where x can be any real number? (without using Calculus)

Solution

$$\begin{aligned}(x^2 - 4x + 3)(x^2 + 4x + 3) &= (x - 3)(x - 1)(x + 3)(x + 1) \\ &= (x^2 - 9)(x^2 - 1) \\ &= (x^2 - 5 - 4)(x^2 - 5 + 4) \\ &= (x^2 - 5)^2 - 16\end{aligned}$$

which has -16 as its minimum value

Alternative approaches

$$\begin{aligned}\text{(i) } \dots (x^2 - 9)(x^2 - 1) &= x^4 - 10x^2 + 9 \\ &= (x^2 - 5)^2 - 16\end{aligned}$$

$$\begin{aligned}\text{(ii) } (x^2 - 4x + 3)(x^2 + 4x + 3) \\ &= x^4 + x^3(4 - 4) + x^2(3 - 16 + 3) + x(-12 + 12) + 9 \\ &= x^4 - 10x^2 + 9 \\ &= (x^2 - 5)^2 - 16\end{aligned}$$