

STEP/Polynomials Q3 (26/6/23)

Factorise $6x^4 - 7x^3 - 26x^2 + 7x + 20$

Solution

$$\text{Let } f(x) = 6x^4 - 7x^3 - 26x^2 + 7x + 20$$

$$f(1) = 6 - 7 - 26 + 7 + 20 = 0, \text{ so that } x - 1 \text{ is a factor}$$

$$f(-1) = 6 + 7 - 26 - 7 + 20 = 0, \text{ so that } x + 1 \text{ is a factor}$$

$$\text{So } f(x) = (x^2 - 1)(6x^2 + ax - 20)$$

$$\text{Equating coeffs of } x: 7 = -a$$

$$\text{Then write } 6x^2 - 7x - 20 = 6x^2 + bx + cx - 20,$$

$$\text{where } bc = 6(-20)$$

One of b & c must be positive, and the other negative.

Suppose WLOG that $b > 0$ & $c < 0$

$$b = 6, c = -20 \Rightarrow b + c = -14$$

$$b = 12, c = -10 \Rightarrow b + c = 2$$

$$b = 10, c = -12 \Rightarrow b + c = -2$$

$$b = 8, c = -15 \Rightarrow b + c = -7$$

$$\text{So } f(x) = 6x^2 - 7x - 20 = 6x^2 + 8x - 15x - 20$$

$$= 2x(3x + 4) - 5(3x + 4) = (2x - 5)(3x + 4)$$

$$\text{and } f(x) = (x - 1)(x + 1)(2x - 5)(3x + 4)$$