STEP/Polynomials: Exercises - Overview (26/6/23)

Q1

Factorise $2x^3 - 33x^2 - 6x + 11$

Q2

Factorise (a) $x^3 - y^3$ (b) $x^3 + y^3$

Q3

Factorise $6x^4 - 7x^3 - 26x^2 + 7x + 20$

Q4

What is the minimum value of $(x^2 - 4x + 3)(x^2 + 4x + 3)$, where x can be any real number? (without using Calculus)

Q5

(i) Find an expansion for $(a + b + c)^3$, and give a justification for the coefficients.

(ii) Extend this to $(a + b + c)^4$

Q6

How many solutions are there to $x^3 - 6x^2 + 9x + 2 = 0$?

Q7

Write out the possible factorisations of $x^n - y^n$ and $x^n + y^n$

Q8

Let $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$,

where $n \geq 2$ and the a_i are integers, with $a_0 \neq 0$.

Suppose that there is a rational root $\frac{p}{q}$, where p & q are integers with no common factor greater than 1 and $q > 0$.

By considering $q^{n-1}f(x)$, show that the root will be an integer.
[From STEP 2011, P3, Q2]