# STEP : Mechanics Notes (4 pages; 20/7/23)

See also notes on the Mechanics page.

## Contents



## (1) Forces determined by motion of a particle

(1.1) In some situations the forces are known, and N2L is used to establish the acceleration of the particle.

In other situations, the motion of a particle is known (eg it is forced to follow a particular path), and this determines the coordinates $x$ & $y$ as functions of time $t$ (or sometimes another parameter; eg an angle $\theta$).

Differentiating $x(t)$ and $y(t)$ twice, then gives the components of acceleration $\ddot{x}(t)$ and $\ddot{y}(t)$.

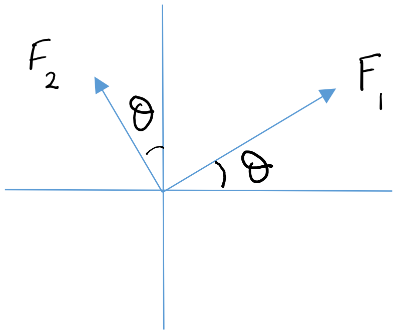
If it convenient to establish forces in the $x$ & $y$ directions, then N2L can be applied straightaway, to give $F_x(t) = m\ddot{x}(t)$ and $F_y(t) = m\ddot{y}(t)$ [noting that the forces will in general vary with time].

If the $x$ & $y$ directions are not convenient, then $\ddot{x}(t)$ and $\ddot{y}(t)$

can be resolved in more suitable directions, to give (for example, referring to the diagram)

$F_1 = m(\ddot{x}(t)cos\theta + \ddot{y}(t)sin\theta)$

and $F_2 = m(-\ddot{x}(t)sin\theta + \ddot{y}(t)cos\theta)$




[See STEP 2021, P3, Q10, where $x$ & $y$ can be determined as functions of an angle $\theta$.]

(1.2) If a particle is known to follow a circular path, then the resultant of the forces directed towards the centre of the circle will equal $\frac{mv^2}{r}$ (where $v$ is the speed of the particle, and $r$ the distance from the centre of the circle).

Note that $v$ and/or $r$ could vary with time, and there could be instantaneous circular motion about a moving centre (with $\frac{mv^2}{r}$ being the instantaneous force towards the instantaneous centre).

[See STEP 2021, P3, Q10.]

**(2) Direction of friction**

One complication that sometimes has to be allowed for is the fact that the direction of friction can switch, as other parameters change. Thus, if a force diagram is set up, the direction of friction, F may need to be provisional (if it isn't obvious from the situation), and will need to be reversed if it turns out that F would otherwise be negative.

## (3) Friction and ladders

In Fig. 1, where a ladder rests against a vertical wall, the ladder is constrained to move up or down the wall. In most cases the attempted motion is down the wall, and so friction acts up the wall. In Fig. 2 however, the ladder can only move initially along the line of the ladder, and so (in most cases) friction acts up the ladder. This means that the normal reaction $R$ has different directions in the two cases. Note that the total reaction force is made up of the two perpendicular components $F$ and $R$ (where $F$ might be zero).

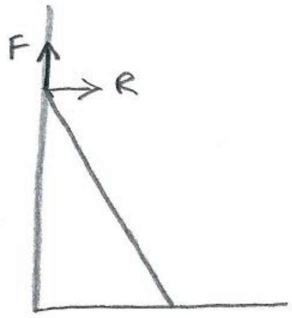



Fig. 1

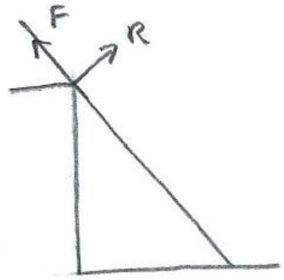



Fig. 2

## (4) Conservation of momentum

Momentum is conserved when there are no external forces, and consequently if objects are colliding on a surface then that surface is usually described as smooth, so that any friction can be ignored. However, one of the A Level exam boards (MEI) includes the following in its Further Maths specification: "The impulse of a finite external force (e.g. friction) acting over a very short period of time (e.g. in a collision) may be regarded as negligible" [as Impulse = Force x Time].

So it is possible in theory for Conservation of Momentum to be invoked (approximately) even when a surface isn't in fact smooth.