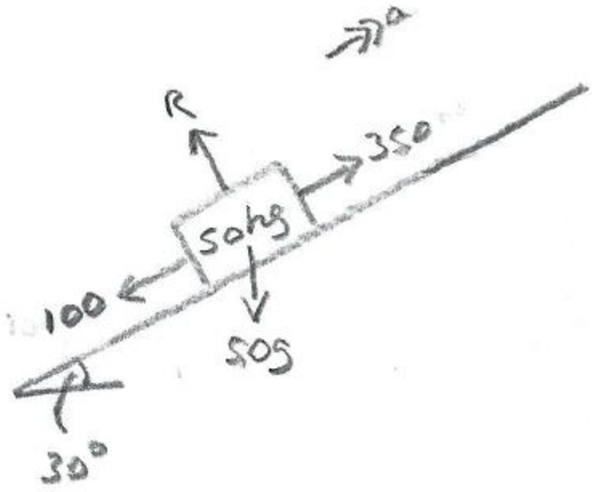
Block on slope



Best directions in which to resolve forces?

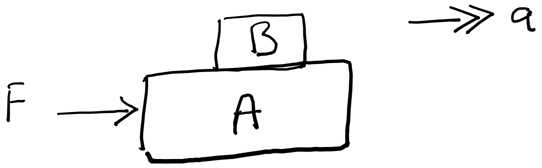
Along and perpendicular to the slope (as there will be no acceleration perpendicular to the slope).

In which direction will friction be acting in the following scenarios?

(i) A block is on the point of slipping down a slope, but is held in place by a force up the slope.

(ii) A block is on the point of slipping up a slope, as a result of a force acting up the slope.

Block on block



A and B have masses M and m respectively.

Block B is not slipping on A

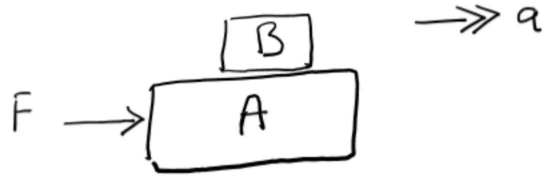
Draw force diagrams and set up N2L for:

(i) A & B combined

(ii) B

(iii) A

Solution



A & B combined: $F = (M + m)a \quad (1)$

B: $F' = ma \quad (2)$,

where F' is the frictional force on B (to the right)

[If there were no friction then B would move to the left, relative to A. So, as friction opposes motion (or attempted motion), it acts to the right.]

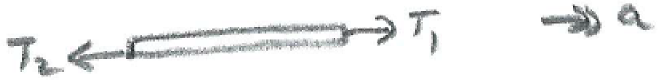
C: $F - F' = Ma \quad (3)$,

where F' is the reaction force of B on A (to the left) (by N3L)

[(2) & (3)$\Rightarrow F - ma = Ma \Rightarrow F = (M + m)a$,

which agrees with (1).]

Tension in a towbar



Why do we assume that $T_1 = T_2$?

By N2L, $T_1 - T_2 = ma$

If the towbar is assumed to have negligible mass, then $T_1 = T_2$ for the purpose of our model.

For a car pulling a trailer, when could there be a compression in the towbar?

Solution

If the car and trailer are decelerating at a rate d, then the tension T in the towbar is given by:

$R_T - T = m_T d$,

where m_T is the mass of the trailer, and R_T is the resistance on it.

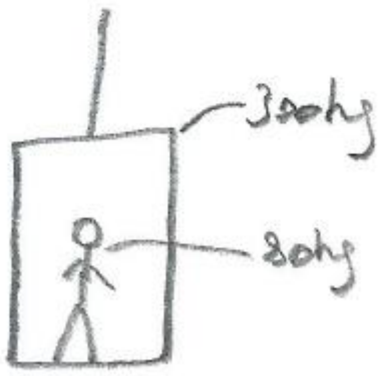
Thus the towbar will be under compression if

$T = R_T - m_T d < 0$; ie if $R_T < m_T d$;

ie if either the deceleration is big enough (due to heavy braking), or if R_T is sufficiently small.

(For large R_T, the trailer would decelerate at a rate greater than d if it were uncoupled from the towbar, and the tension is needed in order to obtain the smaller deceleration of d.)

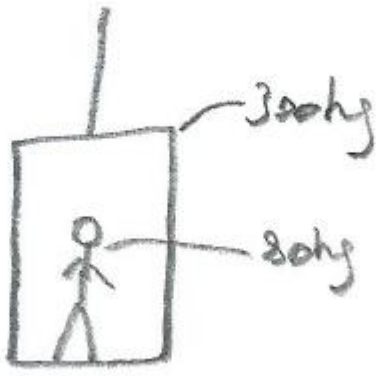
Man in a lift



The lift is accelerating upwards at 0.5 ms^{-2}

Find the tension in the lift cable and the reaction of the man on the floor of the lift.

Solution



Treating the lift and the man as a single object, we can draw a force diagram and apply N2L:

$T - (300 + 80)g = (300 + 80)(0.5)$

$\Rightarrow T = 380(10.3) = 3914\ N$

Then drawing a force diagram for the man:

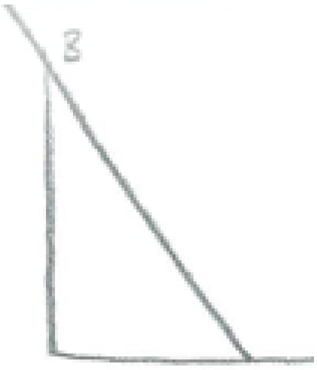
$R - 80g = 80(0.5) \Rightarrow R = 80(10.3) = 824\ N$

This is the reaction of the floor of the lift on the man, but by N3L the reaction of the man on the floor of the lift is also $824\ N$.

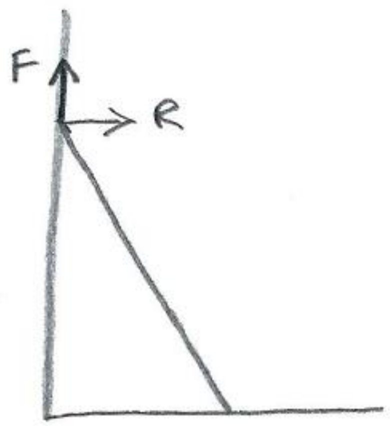
The reaction of the floor of the lift on the man is what the man perceives to be his weight. Compare this with his weight when not accelerating with the lift: $80g = 784\ N$. Thus, when the lift is accelerating upwards, the man feels heavier, compared to his natural weight.

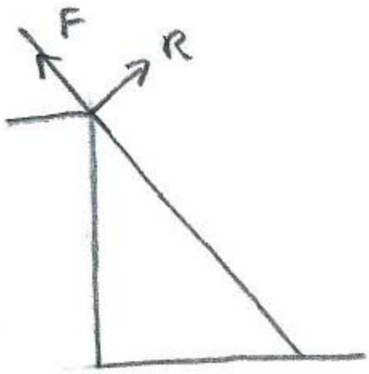
Ladders

Forces acting at A and B?



Solution





A tile slides down a roof (against a known constant frictional force), and then falls to the ground. Choose a method to find:

(a) the speed of the tile when it reaches the foot of the roof

(b) the speed of the tile when it reaches the ground

Solution

A tile slides down a roof (against a known constant frictional force), and then falls to the ground. Choose a method to find:

(a) the speed of the tile when it reaches the foot of the roof

Method A: Loss of PE − gain in KE = work done against friction

Method B: gain in KE = work done by gravity + (negative) work done by friction

Method C: By N2L, $mgsin\theta - F_f = ma$

suvat eq'n: $v^2 = u^2 + 2as \Rightarrow \frac{1}{2}mv^2 = mas$

$= mgsin\theta s - F_f s = mgh -$ work done against friction

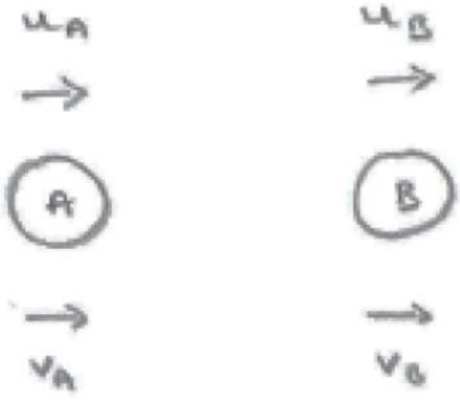
(b) the speed of the tile when it reaches the ground

Method A: Loss of PE = gain in KE, starting from foot of roof

Method B: Loss of PE − gain in KE = work done against friction, starting from top of roof

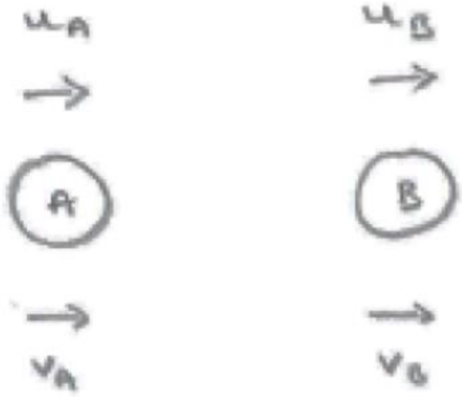
Method C (not advised): Treat tile as projectile from foot of roof

Collisions



If $u_B = 0$ and the masses of A and B are m & km respectively, find the condition necessary for A to change direction.

Solution



CoM: $mu_A = mv_A + kmv_B$

NLR: $v_B - v_A = e(u_A - 0)$

$\Rightarrow u_A = v_A + k(v_A + eu_A)$

$\Rightarrow v_A(1 + k) = u_A(1 - ke)$

$\Rightarrow v_A = \frac{u_A(1-ke)}{(1+k)}$

So $v_A < 0$ when $ke > 1$; ie $e > \frac{1}{k}$ (only possible if $k > 1$).

A rollercaster ride is modelled by a particle on a smooth wire. If a point on the wire has coordinates (x, y), show that

$$\dot{x}\ddot{x} + \dot{y}(\ddot{y} + g) = 0$$

(a) by an energy method, and (b) by applying Newton's 2nd Law

Solution

(a) By Conservation of Energy,

$KE + PE = C$, where C is a constant (as there are no forces other than gravity that do any work)

ie $\frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mgy = C$, where m is the mass of the particle.

Differentiating wrt t, $m(\dot{x}\ddot{x} + \dot{y}\ddot{y}) + mg\dot{y} = 0$,

so that $\dot{x}\ddot{x} + \dot{y}(\ddot{y} + g) = 0$, as required.

(b) As the wire is smooth, the only force on the particle affecting its motion is the component of its weight along the wire.

By N2L, $-mgsin\theta = m(\ddot{x}cos\theta + \ddot{y}sin\theta)$, [see Note below]

where the gradient of the wire is $\frac{dy}{dx} = tan\theta$ at the point (x, y), and $\ddot{x}$ & $\ddot{y}$ are the x & y components of the acceleration of the particle.

Then $-gtan\theta = \ddot{x} + \ddot{y}tan\theta$,

and hence, since $tan\theta = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\dot{y}}{\dot{x}}$,

$-g\dot{y} = \dot{x}\ddot{x} + \ddot{y}\dot{y}$, or $\dot{x}\ddot{x} + \dot{y}(\ddot{y} + g) = 0$, as required.

[Note: If instead the force along the wire is resolved in the x & y directions:

$(-mgsin\theta)cos\theta = m\ddot{x}$ and $(-mgsin\theta)sin\theta = m\ddot{y}$

Then $m(\ddot{x}cos\theta + \ddot{y}sin\theta) = (-mgsin\theta)(cos^2\theta + sin^2\theta)$

$= -mgsin\theta$, as before.]