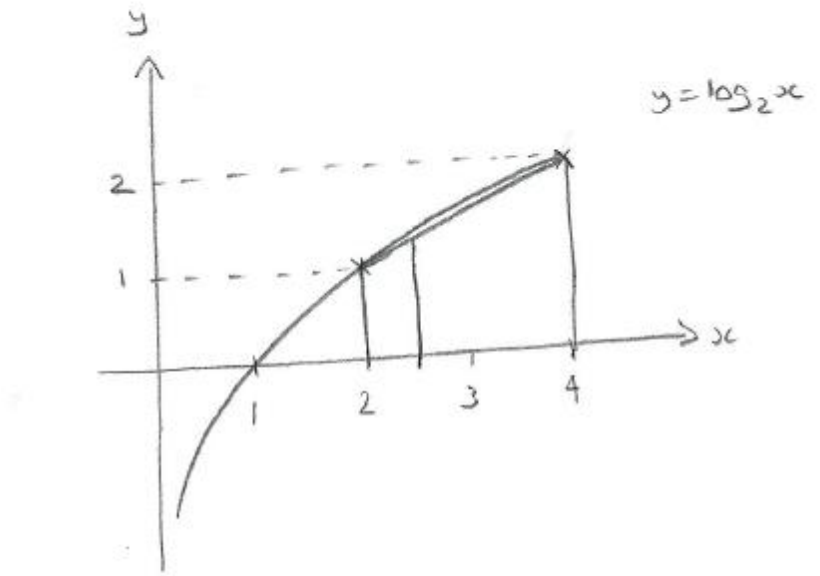
STEP/Logarithms Q4 (24/6/23)

By approximating the graph of

$y = log_2 x$ by a straight line between $x = 2$ and $x = 4$, find an approximate value for $log_2\left(\frac{5}{2}\right)$

Solution



Approach 1: weighted average

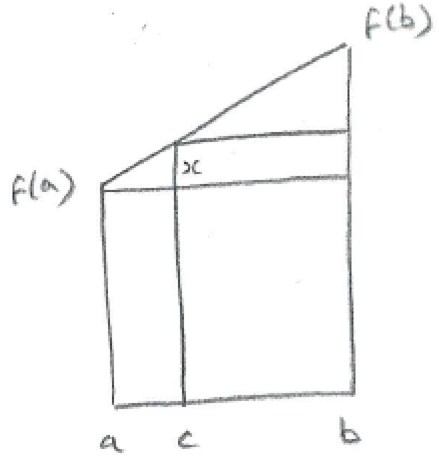
$$log_2\left(\frac{5}{2}\right) \approx \left(\frac{4-2.5}{4-2}\right) log_2 2 + \left(\frac{2.5-2}{4-2}\right) log_2 4$$

$$= (0.75)(1) + (0.25)(2) = 1.25$$

Approach 2: similar triangles

Referring to the diagram below (for the general function $f(x)$)

$$\frac{x}{c-a} = \frac{f(b)-f(a)}{b-a}$$



For our example,

$\frac{x}{2.5-2} = \frac{2-1}{4-2}$,

so that $x = (0.5)(0.5) = 0.25$, and hence $log_2\left(\frac{5}{2}\right) \approx 1 + 0.25 = 1.25$

Approach 3: Equation of line

The gradient of the line is $\left(\frac{f(b)-f(a)}{b-a}\right)$

Then $f(c) \approx f(a) + \left(\frac{f(b)-f(a)}{b-a}\right)(c-a)$

In this case, $log_2\left(\frac{5}{2}\right) \approx 1 + \left(\frac{2-1}{4-2}\right)(2.5-2) = 1.25$ again.

Also $f(a) + \left(\frac{f(b)-f(a)}{b-a}\right)(c-a)$

$= \left(\frac{1}{b-a}\right)\left((b-a)f(a) + (c-a)f(b) - (c-a)f(a)\right)$

$= \left(\frac{1}{b-a}\right)\left((b-c)f(a) + (c-a)f(b)\right)$,

which is the weighted average approach