STEP/Logarithms Q1 (24/6/23)

(i) Show that $log_2 3 > \frac{3}{2}$

(ii) Find an upper bound for $log_2 3$ (as small as possible)

Solution

(i) $log_2 3 > \frac{3}{2} \Leftrightarrow 3 > 2^{\frac{3}{2}}$ (as $y = 2^x$ is an increasing function)

$\Leftrightarrow 3^2 > 2^3$

(ii) Suppose that $log_2 3 < \frac{m}{n}$

Then $3 < 2^{\left(\frac{m}{n}\right)}$ and $3^n < 2^m$

As $243 = 3^5 < 2^8 = 256$, $log_2 3 < \frac{8}{5}$

[and $\frac{8}{5}$ is a reasonably low upper bound, as 243 & 256 are reasonably close]