STEP/Integration Q9 (21/6/23)

$$\int \frac{1}{1+cosx}\,dx$$

Solution

$\int \frac{1}{1+cosx}\ dx = \int \frac{1-cosx}{1-cos^2x}\ dx = \int \frac{1-cosx}{sin^2x}\ dx$

$= \int cosec^2x\, dx - \int \frac{cosx}{sin^2x}\ dx$

Now, as $\frac{d}{dx}tanx = sec^2x$, we can expect $\frac{d}{dx}cotx = a.cosec^2x$,

where $a = 1$ or -1.

To investigate this, $\frac{d}{dx}\ (tanx)^{-1} = -(tanx)^{-2}sec^2x = -\ cosec^2x$

For the 2nd integral, as the integral of the numerator $cosx$ features simply in the rest of the integrand (ie $\frac{1}{sin^2x}$ can be written as $\frac{1}{u^2}$, where $u = sinx$, and $\frac{1}{u^2}$ can easily be integrated), $u = sinx$ leads to $\int \frac{cosx}{sin^2x}\ dx = -\frac{1}{sinx} = -cosecx$

So $\int \frac{1}{1+cosx}\ dx = -cotx + cosecx + c$