

STEP/Integration Q10 (21/6/23)

$$\int \frac{1}{x \ln x} dx$$

Solution

We could try to do this by Parts:

Differentiating $\frac{1}{\ln x}$:

$$\begin{aligned}\int \frac{1}{x \ln x} dx &= \ln x \cdot \frac{1}{\ln x} - \int \ln x \cdot (-1)(\ln x)^{-2} \left(\frac{1}{x}\right) dx \\ &= 1 + \int \frac{1}{x \ln x} dx \quad ?!\end{aligned}$$

[The apparent contradiction here is explained by the constant of integration.]

$$\text{Instead: } \int \frac{1}{x \ln x} dx = \int \frac{\left(\frac{1}{x}\right)}{\ln x} dx, \text{ and } \int \frac{1}{x} dx = \ln x,$$

$$\text{so let } u = \ln x; du = \frac{1}{x} dx,$$

$$\text{and } \int \frac{1}{x \ln x} dx = \int \frac{1}{u} du = \ln(\ln x) + c$$