STEP/Integers Q6 (21/6/23)

Show that numbers of the form $4(n-1)^2+2$ can never be one more than a multiple of 3, where n is a positive integer.

Solution

Case 1: $n = 3p$ (where $p \in \mathbb{Z}^+ \ or\ 0$)

$4(n-1)^2 + 2 = 4(3p-1)^2 + 2 \equiv 4(1) + 2 \text{ (mod 3)} \equiv 0$

[as $4(3p)^2 + 4(2)(3p)(-1)$ is a multiple of 3]

Case 2: $n = 3p + 1$

$4(n-1)^2 + 2 = 4(3p)^2 + 2 \equiv 2$

Case 3: $n = 3p + 2$

$4(n-1)^2 + 2 = 4(3p+1)^2 + 2 \equiv 4 + 2 \ \equiv 0$

So $4(n-1)^2 + 2$ is always $\equiv 0 \ or\ 2$; ie never one more than a multiple of 3.