STEP/Integers Q4 (21/6/23)

Prove that there are no positive integers m and n such that $m^2 = n^2 + 1$

Solution

[Proof by contradiction]

Suppose that $m^2 = n^2 + 1$, where m and n are positive integers.

Then $m^2 - n^2 = 1$,

and hence $(m - n)(m + n) = 1$

As m and n are integers, $m - n$ and $m + n$ will also be integers, and so they are either both 1 or both -1

But $m + n > 0$, so that $m - n = 1$ and $m + n = 1$

Subtracting the 1st eq'n from the 2nd gives $2n = 0$, so that $n = 0$, which contradicts the assumption that n is a positive integer.

So there are no positive integers m and n such that $m^2 = n^2 + 1$