

STEP/Integers Q3 (21/6/23)

Show that $3^{57} - 2^{57}$ cannot be prime.

Solution

We could consider using the result

$$x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \dots + xy^{n-2} + y^{n-1})$$

but it isn't of any use having $x - y = 3 - 2 = 1$.

However, we can write $3^{57} - 2^{57}$ as $(3^{19})^3 - (2^{19})^3$, for example, to give the factor $3^{19} - 2^{19}$ (or writing it instead as

$(3^3)^{19} - (2^3)^{19}$, $3^3 - 2^3$ is also seen to be a factor).

So $3^{57} - 2^{57}$ isn't a prime number.