STEP/Integers Q2 (21/6/23)

Find all positive integer solutions of the equation

$xy - 8x + 6y = 90$

Solution

[Aiming for something of the form $f(x)g(y) = c$, where c is an integer:]

$$xy - 8x + 6y = (x + 6)(y - 8) + 48,$$

so that the original equation is equivalent to

$$(x + 6)(y - 8) = 42$$

The positive integer solutions are given by:

$$x + 6 = 7, y - 8 = 6$$

$$x + 6 = 14, y - 8 = 3$$

$$x + 6 = 21 \; y - 8 = 2$$

$$x + 6 = 42, y - 8 = 1,$$

so that the solutions are:

$$x = 1, y = 14$$

$$x = 8, y = 11$$

$$x = 15, y = 10$$

$$x = 36, y = 9$$