STEP/Integers Q1 (21/6/23)

Can n^3 equal $n + 12345670$ (where n is a positive integer)?

Solution

Rearrange to $n^3 - n = 12345670$

$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$, and one of these factors must be a multiple of 3; whereas 12345670 is not a multiple of 3 (since $1 + 2 + 3 + 4 + 5 + 6 + 7 + 0$ isn't a multiple of 3); so answer is No.