STEP/Integers: Exercises - Overview (19/2/24)

Q1

Can n^3 equal $n + 12345670$ (where n is a positive integer)?

Q2

Find all positive integer solutions of the equation

$$xy - 8x + 6y = 90$$

Q3

Show that $3^{57} - 2^{57}$ cannot be prime.

Q4

Prove that there are no positive integers m and n such that
$m^2 = n^2 + 1$

Q5

Show that the product of 4 consecutive positive integers is never a perfect square.

Q6

Show that numbers of the form $4(n - 1)^2 + 2$ can never be one more than a multiple of 3, where n is a positive integer.

Q7

Let $h(a, b)$ denote the highest common factor of a & b. Suppose that $b = ka + r$, where k, a & r are positive integers.

Prove that $h(a, b) = h(a, r)$.

Q8

Let $f(n)$ be the number of factors, other than 1, of the number n.

Show that, if m & n have no common factors,

then $f(mn) = f(m)f(n) + f(m) + f(n)$