STEP/Inequalities Q7 (20/6/23)

Prove or provide a counter-example for the conjecture

$x > a$ & $y > b \Rightarrow xy > ab$ ($a, b \; real$) in each of the following cases:

(i) $a > 0, b > 0$ (ii) $a < 0, b < 0$ (iii) $a > 0, b < 0$

Solution

(i) $x > a \Rightarrow xy > ay$ [as $y > 0$] $> ab$ [since $y > b \Rightarrow ay > ab$]

so true

[or refer to graph of $y = ab$]

(ii) $a < 0, b < 0$

counter-example: $x = 0$

(iii) $a > 0, b < 0$

consider graph of $xy = ab$ when $a = 3, b = -2$ (see below)

(counter-example: $x = 4 + \delta, y = -2 + \delta$)