STEP/Inequalities Q2 (20/6/23)

Assuming that $sin^2\theta + cos^2\theta = 1$, but without using any compound angle results, show that $sin\theta cos\theta \leq \frac{1}{2}$

Solution

$$(sin\theta - cos\theta)^2 \geq 0 \Rightarrow sin^2\theta + cos^2\theta - 2sin\theta cos\theta \geq 0$$

$$\Rightarrow 1 \geq 2sin\theta cos\theta \Rightarrow sin\theta cos\theta \leq \frac{1}{2}$$