STEP/Inequalities Q11 (20/6/23)

Given that $p, q > 0$ and that $p \neq q$, show that

$p^{2n}q + q^{2n}p > (pq)^n q + (qp)^n p$

Solution

$p^{2n}q + q^{2n}p - (pq)^n q - (qp)^n p$

$= p^n q(p^n - q^n) + q^n p(q^n - p^n)$

$= (p^n - q^n)(p^n q - q^n p)$

$= (p^n - q^n)pq(p^{n-1} - q^{n-1})$

$= (p - q)(p^{n-1} + qp^{n-2} + \cdots + q^{n-1})pq$

$. (p - q)(p^{n-2} + qp^{n-3} + \cdots + q^{n-2})$

> 0, as $(p - q)^2 > 0$ (as $p \neq q$)