STEP/Hyperbolic Functions Q3 (16/6/23)

Simplify $sinh(arcoshx)$ & $cosh(arsinhx)$

Solution

$sinh(arcoshx) = sinh\ (ln\ (x + \sqrt{x^2 - 1})$

$= \frac{1}{2}([x + \sqrt{x^2 - 1}] - [\frac{1}{x+\sqrt{x^2-1}}])$

$= \frac{1}{2}([x + \sqrt{x^2 - 1}] - [\frac{1}{x+\sqrt{x^2-1}}][\frac{x-\sqrt{x^2-1}}{x-\sqrt{x^2-1}}])$

$= \frac{1}{2}([x + \sqrt{x^2 - 1}] - \frac{[x-\sqrt{x^2-1}]}{1})$

$= \sqrt{x^2 - 1}$

$cosh(arsinhx) = cosh\ (ln\ (x + \sqrt{x^2 + 1})$

$= \frac{1}{2}([x + \sqrt{x^2 + 1}] + [\frac{1}{x+\sqrt{x^2+1}}])$

$= \frac{1}{2}([x + \sqrt{x^2 + 1}] + [\frac{1}{x+\sqrt{x^2+1}}][\frac{x-\sqrt{x^2+1}}{x-\sqrt{x^2+1}}])$

$= \frac{1}{2}([x + \sqrt{x^2 + 1}] + \frac{[x-\sqrt{x^2+1}]}{-1})$

$= \sqrt{x^2 + 1}$