

STEP/Hyperbolic Functions: Exercises - Overview

(16/6/23)

Q1

Simplify $\sinh(\cosh^{-1}2)$

Q2

Given that $\operatorname{artanh}x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$ and $\operatorname{arcoth}x = \frac{1}{2} \ln\left(\frac{1+x}{x-1}\right)$,

and also that $\frac{d}{dx}(\operatorname{artanh}x) = \frac{d}{dx}(\operatorname{arcoth}x) = \frac{1}{1-x^2}$,

what is wrong with the following reasoning?

$$\int \frac{1}{1-x^2} dx = \operatorname{artanh}x + C = \operatorname{arcoth}x + C_1,$$

so that $\operatorname{artanh}x - \operatorname{arcoth}x = C_2$

$$\text{But } \operatorname{artanh}x - \operatorname{arcoth}x = \frac{1}{2} \ln\left(\frac{\frac{1+x}{1-x}}{\frac{1+x}{x-1}}\right) = \frac{1}{2} \ln\left(\frac{x-1}{1-x}\right) = \frac{1}{2} \ln(-1),$$

which isn't defined!

Q3

Simplify $\sinh(\operatorname{arcosh}x)$ & $\cosh(\operatorname{arsinh}x)$

Q4

Given that $\sinh x = \tanh y$, where $-\frac{\pi}{2} < y < \frac{\pi}{2}$, show that

(a) $\tanh x = \sin y$ (b) $x = \ln(\tanh y + \sec y)$