

Hyperbolic Functions

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Solution

$$\begin{aligned}\sinh(4u) &= 2 \sinh(2u) \cosh(2u) \\ &= 4 \sinh u \cosh u (\cosh^2 u + \sinh^2 u) \\ &= 4x \sqrt{1+x^2} (1+2x^2)\end{aligned}$$

Show that $\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ ($|x| < 1$)

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If $y = \operatorname{artanh} x$, then $\operatorname{tanh} y = x$ ($|x| < 1$)

$$\Rightarrow x = \frac{\frac{1}{2}(e^y - e^{-y})}{\frac{1}{2}(e^y + e^{-y})} = \frac{e^{2y} - 1}{e^{2y} + 1}$$

$$\Rightarrow x(e^{2y} + 1) = e^{2y} - 1$$

$$\Rightarrow e^{2y}(x - 1) = -1 - x$$

$$\Rightarrow e^{2y} = \frac{1+x}{1-x}$$

$$\Rightarrow y = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$