

STEP/General Q2 (13/6/23)

Find the square roots of $49 - 12\sqrt{5}$

Solution

$$\text{Let } x^2 = 49 - 12\sqrt{5}$$

$$\text{Consider } x = a + b\sqrt{5}$$

$$\text{Then } a^2 + 2ab\sqrt{5} + 5b^2 = 49 - 12\sqrt{5}$$

$$\text{Let } a^2 + 5b^2 = 49 \text{ \& } 2ab = -12$$

[a variation on Equating Coefficients]

Looking for integer solutions, we see that either

$a = 2 \text{ \& } b = -3$ or $a = -2 \text{ \& } b = 3$ work.