STEP/Forces, Q6 (13/6/23)

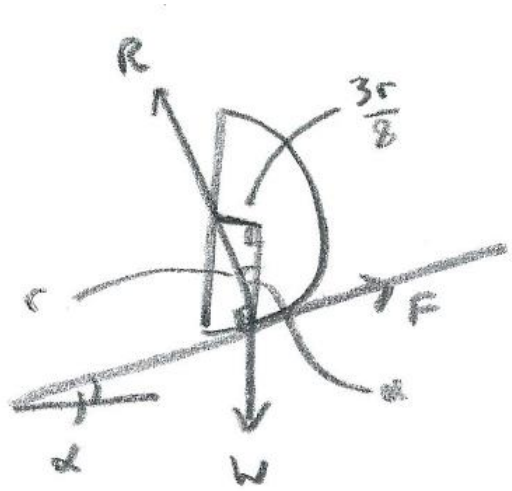
A uniform solid hemisphere rests in equilibrium on a rough slope, with its curved surface in contact with the slope, which is inclined at an angle α to the horizontal, in such a way that the plane face of the hemisphere is vertical. Find α.

Solution

In order to establish the necessary configuration of the hemisphere and slope, we note that the weight of the hemisphere must act on a line that passes through the point of contact between the hemisphere and the slope.

[The three forces acting on the hemisphere (its weight, the reaction from the slope and friction) will then all meet at a single point, as is required for a body in equilibrium that is subject to three forces - otherwise a non-zero moment would exist about the point of intersection of two of the forces.]

A diagram can be drawn by starting with the hemisphere, and adding in the slope. Note also that the point of contact will be on a tangent to the hemisphere, and that the perpendicular to the tangent, along which the reaction force acts, will be a radius of the hemisphere.



The radius of the hemisphere (which we are expecting to cancel out, as it isn't mentioned in the question) can be taken to be r.

The weight can be taken to act at the centre of mass of the hemisphere, which is at a distance $\frac{3r}{8}$ from the plane face.

From the diagram, $sin\alpha = \frac{\left(\frac{3r}{8}\right)}{r} = \frac{3}{8}$, and hence $\alpha = 22.0°$ (1dp).