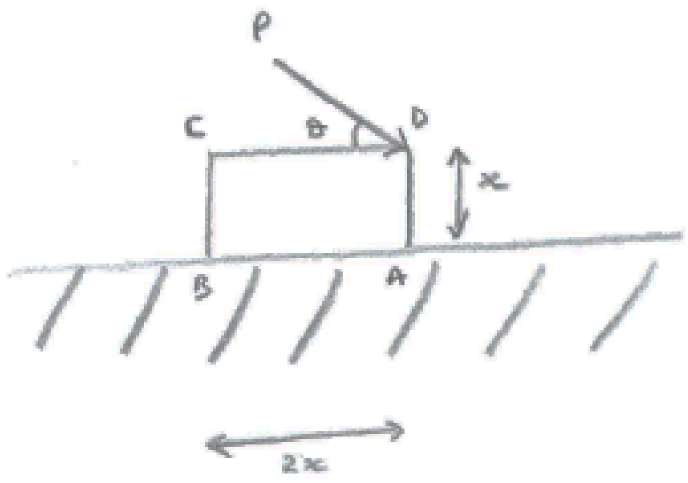
STEP/Forces, Q4 (11/6/23)

A uniform block of mass m rests on a table, and a force P is applied at D, as shown in the diagram. The block has length $2x$ and height x. The coefficient of friction between the block and the table is μ.



(i) If the block is on the point of sliding, find an expression for P.

(ii) If instead the block is on the point of toppling, find an expression for P.

(iii) If the block is to topple before it slides, find a condition on μ.

Solution

(i) The normal reaction, $R = mg + Psin\theta$

The frictional force $= \mu(mg + Psin\theta)$

Hence, at the point of sliding, $\mu(mg + Psin\theta) = Pcos\theta$,

so that $P(cos\theta - \mu sin\theta) = \mu mg$

and $P = \frac{\mu mg}{cos\theta - \mu sin\theta}$

(ii) If the block is on the point of toppling, it will be about A, and the only reaction on the block will be at A. [This will be a combination of a normal reaction and friction.]

As the block is uniform, its weight will act at a distance x from AD, and so, taking moments about A,

$(mg)x = (Pcos\theta)x$

[the normal reaction and friction contribute nothing, as they act at A]

Hence $P = \frac{mg}{cos\theta}$

(iii) At the critical position where the block is about to both slide and topple,

$P = \frac{\mu mg}{cos\theta - \mu sin\theta} = \frac{mg}{cos\theta}$

so that $\mu cos\theta = cos\theta - \mu sin\theta$;

$\mu(cos\theta + sin\theta) = cos\theta$

and $\mu = \frac{cos\theta}{cos\theta + sin\theta} = \frac{1}{1 + tan\theta}$

So, if the block is to topple before it slides, we require

$\mu > \frac{1}{1+tan\theta}$ [ie making the frictional force greater]