STEP/Forces, Q3 (11/6/23)

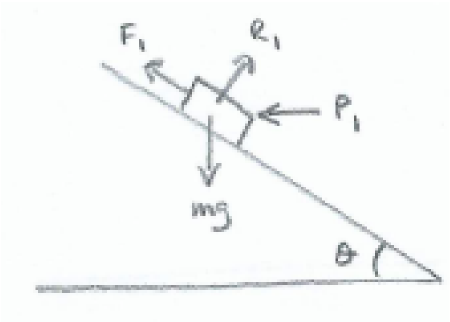
A block rests on a slope which is angled at $\theta°$ to the horizontal. The coefficient of friction between the surface of the slope and the block is $tan\ \alpha$. P_1 is the horizontal force that needs to be applied to the block to stop it from slipping down the slope, whilst P_2 is the greatest horizontal force that can be applied without the block slipping up the slope.

(i) Show that $\frac{P_2}{P_1} = \frac{\tan(\theta+\alpha)}{\tan(\theta-\alpha)}$

(ii) Explain what happens when $\theta < \alpha$

Solution

(i) As friction acts to oppose attempted motion, the frictional force acts up the slope in the first case, and down in the second.



In the first case, resolving forces perpendicular to the slope,

$N2L \Rightarrow R_1 = mgcos\theta + P_1 sin\theta$

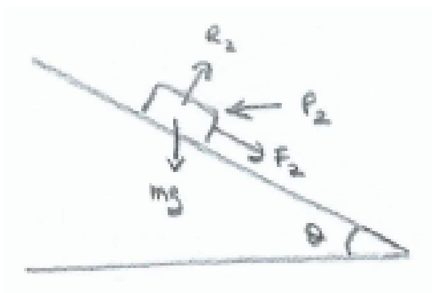
Limiting friction $\Rightarrow F_1 = tan\alpha R_1$

Resolving forces along the slope,

$N2L \Rightarrow F_1 + P_1 cos\theta = mgsin\theta$

Hence $tan\alpha(mgcos\theta + P_1 sin\theta) + P_1 cos\theta = mgsin\theta$

and so $P_1(cos\theta + tan\alpha sin\theta) = mg(sin\theta - tan\alpha cos\theta)$ (1)



In the second case, $R_2 = mgcos\theta + P_2 sin\theta$, $F_2 = tan\alpha R_2$

and $F_2 + mgsin\theta = P_2 cos\theta$

Hence $tan\alpha(mgcos\theta + P_2sin\theta) + mgsin\theta = P_2cos\theta$

and so $P_2(cos\theta - tan\alpha sin\theta) = mg(sin\theta + tan\alpha cos\theta)$ (2)

Then (1) & (2) $\Rightarrow \frac{P_2}{P_1} = \frac{(sin\theta + tan\alpha cos\theta)(cos\theta + tan\alpha sin\theta)}{(cos\theta - tan\alpha sin\theta)(sin\theta - tan\alpha cos\theta)}$

$= \frac{(tan\theta + tan\alpha)(1 + tan\alpha tan\theta)}{(1 - tan\alpha tan\theta)(tan\theta - tan\alpha)} = \frac{\tan(\theta + \alpha)}{\tan(\theta - \alpha)}$

(ii) $tan\theta$ is the minimum value of μ required for the block to rest on the slope without slipping. Hence, when $\theta < \alpha$, so that $tan\theta < tan\alpha = \mu$, no horizontal force is needed.