STEP/Forces: Exercises - Overview (13/6/23)

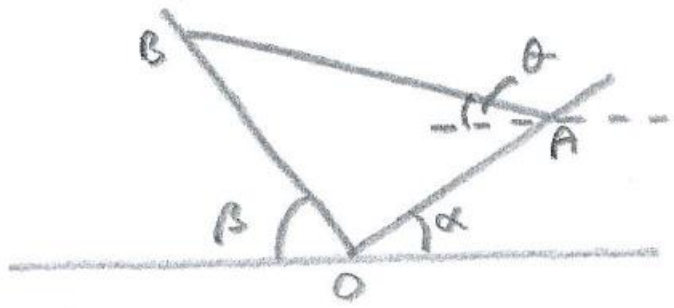
Q1

A uniform rod AB lies in equilibrium between two smooth planes inclined at angles α and β to the horizontal, as shown in the diagram, where $\beta > \alpha$, such that the vertical plane containing AB is perpendicular to the line of intersection of the two planes.

(i) Show that the ratio of the reactions at A and B is $sin\beta: sin\alpha$

(ii) If AB makes an angle θ to the horizontal, show that

$tan\theta = \frac{\sin(\beta-\alpha)}{2sin\alpha sin\beta}$



Q2

(i) A lift of mass $400kg$ has a maximum acceleration or deceleration of $1ms^{-2}$, and the lift cable can support a tension of $9000N$. What is the maximum number of people of mass $80kg$ that can safely be carried? (Assume $g = 10ms^{-2}$.)

(ii) If a single person of mass $80kg$ is in the lift when it is accelerating downwards at $1ms^{-2}$, how much lighter does the person feel, compared with their usual weight, as a percentage?

Q3

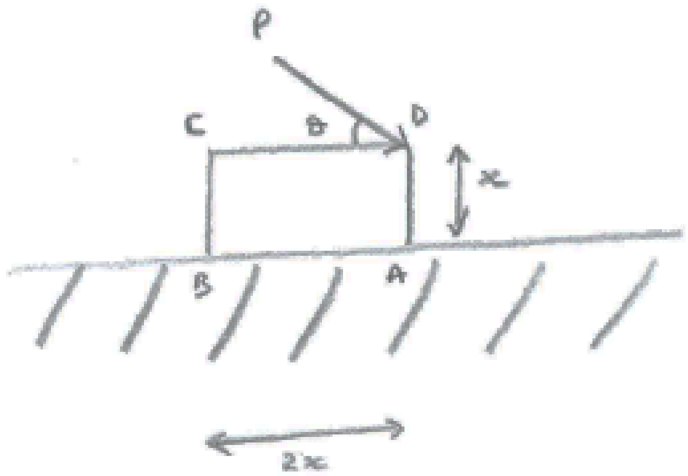
A block rests on a slope which is angled at $\theta°$ to the horizontal. The coefficient of friction between the surface of the slope and the block is $tan\ \alpha$. P_1 is the horizontal force that needs to be applied to the block to stop it from slipping down the slope, whilst P_2 is the greatest horizontal force that can be applied without the block slipping up the slope.

(i) Show that $\frac{P_2}{P_1} = \frac{\tan(\theta+\alpha)}{\tan(\theta-\alpha)}$

(ii) Explain what happens when $\theta < \alpha$

Q4

A uniform block of mass m rests on a table, and a force P is applied at D, as shown in the diagram. The block has length $2x$ and height x. The coefficient of friction between the block and the table is μ.



(i) If the block is on the point of sliding, find an expression for P.

(ii) If instead the block is on the point of toppling, find an expression for P.

(iii) If the block is to topple before it slides, find a condition on μ.

Q5

A rollercaster ride is modelled by a particle on a smooth wire. If a point on the wire has coordinates (x, y), show that

$\dot{x}\ddot{x} + \dot{y}(\ddot{y} + g) = 0$

(a) by an energy method, and (b)(as an alternative method)

by applying Newton's 2nd Law

Q6

A uniform solid hemisphere rests in equilibrium on a rough slope, with its curved surface in contact with the slope, which is inclined at an angle α to the horizontal, in such a way that the plane face of the hemisphere is vertical. Find α.

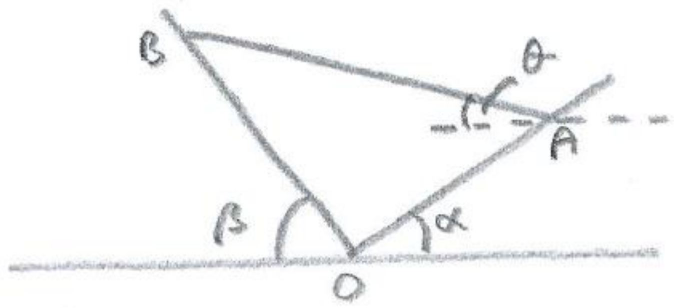
Q7

A uniform rod AB lies in equilibrium between two smooth planes inclined at angles α and β to the horizontal, as shown in the diagram, where $\beta > \alpha$, such that the vertical plane containing AB is perpendicular to the line of intersection of the two planes.

(i) Show that the ratio of the reactions at A and B is $sin\beta : sin\alpha$

(ii) If AB makes an angle θ to the horizontal, show that

$tan\theta = \frac{\sin(\beta-\alpha)}{2sin\alpha sin\beta}$



[from Wragg: "Modern Mechanics - A vectorial approach"]