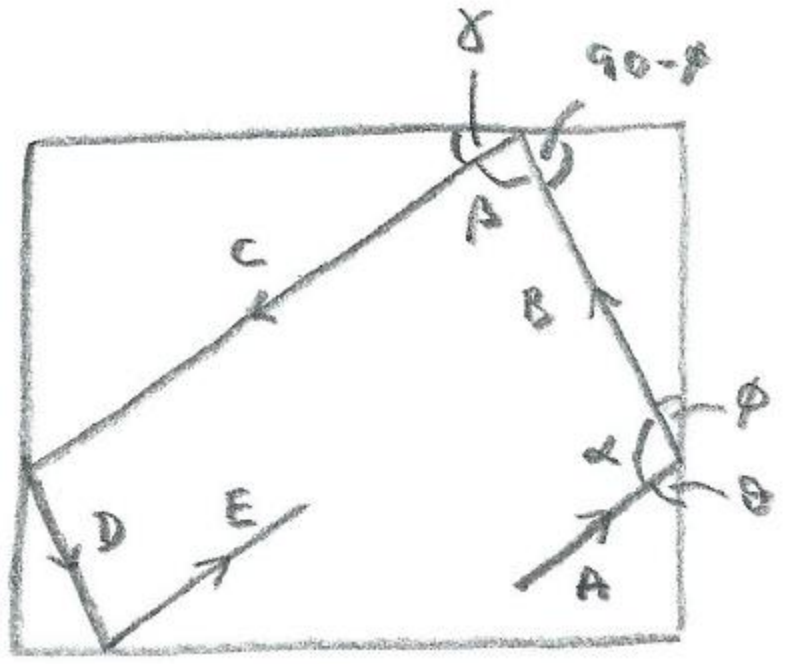
STEP, Collisions – Q8 (11/6/23)

A snooker ball is hit towards a cushion, with speed v, in such a way that it hits each of the four sides of the table. The coefficient of restitution between the ball and the cushions is e. Investigate the speed and direction of the ball.

Solution



(a)(i) Referring to the diagram, when the ball is at A (travelling towards the 1st cushion), its velocity vector is $\begin{pmatrix} vsin\theta \\ vcos\theta \end{pmatrix}$, and the gradient of its path is $cot\theta$.

(ii) When the ball is at B (travelling towards the 2nd cushion), its velocity vector is $\begin{pmatrix} -evsin\theta \\ vcos\theta \end{pmatrix}$, and the gradient of its path is $-\frac{1}{e}cot\theta$.

(iii) To find the relation between θ and ϕ:

(a) See note on Oblique impacts, which shows that $tan\phi = etan\theta$

(b) This can be verified by considering the slope at B:

$\tan\phi = \frac{evsin\theta}{vcos\theta} = etan\theta$

(c) A more complicated approach is:

$$cos\phi = \frac{\begin{pmatrix} -evsin\theta \\ vcos\theta \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{v\sqrt{e^2sin^2\theta + cos^2\theta}(1)} = \frac{vcos\theta}{v\sqrt{e^2sin^2\theta + cos^2\theta}} = \frac{cos\theta}{\sqrt{e^2sin^2\theta + cos^2\theta}}$$

$$\Rightarrow cos^2\phi = \frac{cos^2\theta}{e^2sin^2\theta + cos^2\theta} = \frac{1}{e^2tan^2\theta + 1}$$

$$\Rightarrow e^2tan^2\theta + 1 = sec^2\phi = tan^2\phi + 1$$

$$\Rightarrow e^2tan^2\theta = tan^2\phi$$

$\Rightarrow tan\phi = etan\theta$ (as $e > 0$ and $\theta, \phi < 90°$)

(iv) When the ball is at C (travelling towards the 3rd cushion), its velocity vector is $\begin{pmatrix} -evsin\theta \\ -evcos\theta \end{pmatrix}$, and the gradient of its path is $cot\theta$.

So the path at C is parallel to that at A; ie it has turned through 180°.

It follows that $\alpha + \beta = 180°$ (from the properties of parallel lines).

(v) The speed of the ball at C is $\sqrt{(-evsin\theta)^2 + (-evcos\theta)^2}$

$= ev$

(vi) To find an expression for γ:

$$\gamma + \beta + (90 - \phi) = 180$$

$$\Rightarrow \gamma = 90 - \beta + \phi = 90 - (180 - \alpha) + \phi$$

$$= \alpha + \phi - 90$$

$$= (180 - \theta - \phi) + \phi - 90$$

$= 90 - \theta$

(vii) When the ball is at D (travelling towards the 4th cushion), its velocity vector is $\begin{pmatrix} e^2 v sin\theta \\ -ev cos\theta \end{pmatrix}$, and the gradient of its path is $-\frac{1}{e} cot\theta$. So the path at D is parallel to that at B.

(viii) When the ball is at E (travelling away from the 4th cushion), its velocity vector is $\begin{pmatrix} e^2 v sin\theta \\ e^2 v cos\theta \end{pmatrix}$, and the gradient of its path is $cot\theta$. So the path at E is parallel to that at A.

(ix) The speed of the ball at E is $e^2 v$.