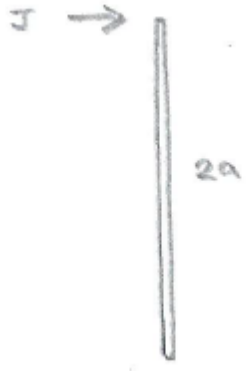
STEP/Collisions - Q7 (11/6/23)

An impulse J is applied to one end of a thin, uniform rod of length $2a$ and mass m, as shown below. Describe the resulting motion.



Solution

By conservation of linear momentum, if v is the velocity of the centre of mass of the rod after the impulse, then:

$J = mv \quad (1)$

And by conservation of angular momentum, if ω is the angular velocity about the centre of mass after the impulse, then

$aJ = I\omega \quad (2)$,

where I, the moment of inertia of the rod about an axis through the centre of mass, perpendicular to the rod $= \frac{1}{3}ma^2$

So, the motion of the rod after the impulse is a combination of a velocity of $v = \frac{J}{m}$ in the direction of the impulse, together with a rotation about the centre of mass, with angular velocity

$$\omega = \frac{aJ}{\left(\frac{1}{3}ma^2\right)}$$

$$= \frac{3J}{ma}$$