STEP, Collisions – Q6 (11/6/23)

Ball A of mass m, travelling with speed u on a smooth surface, collides directly with ball B of mass km, which is at rest. The coefficient of restitution between the two balls is e.

Show that the loss of kinetic energy is greatest when $e = 0$.

Solution

Let v_A & v_B be the final speeds of A & B in the original direction of A.

By conservation of momentum, $mu = mv_A + kmv_B$,

so that $u = v_A + kv_B$

And by Newton's law of restitution, $v_B - v_A = eu$

Adding these eq'ns then gives $v_B = \frac{u(e+1)}{(k+1)}$

and $v_A = \frac{u(e+1)}{(k+1)} - eu = \frac{u}{(k+1)}\left(e + 1 - e(k+1)\right) = \frac{u(1-ek)}{(k+1)}$

The loss of kinetic energy is $\frac{1}{2}mu^2 - \frac{1}{2}m{v_A}^2 - \frac{1}{2}km{v_B}^2$

Thus the loss will be greatest when $u^2 - {v_A}^2 - k{v_B}^2$

$= u^2\{1 - \frac{(1-ek)^2}{(k+1)^2} - \frac{k(e+1)^2}{(k+1)^2}\}$ is greatest;

ie when $(k+1)^2 - (1-ek)^2 - k(e+1)^2$ is greatest,

This expression equals $k^2 + 2k - e^2k^2 + 2ek - ke^2 - 2ke - k$

$= k^2 + k - e^2k^2 - ke^2$

$= (k^2 + k)(1 - e^2)$, which is maximised when $e = 0$

[It also shows that no kinetic energy is lost when $e = 1$.]